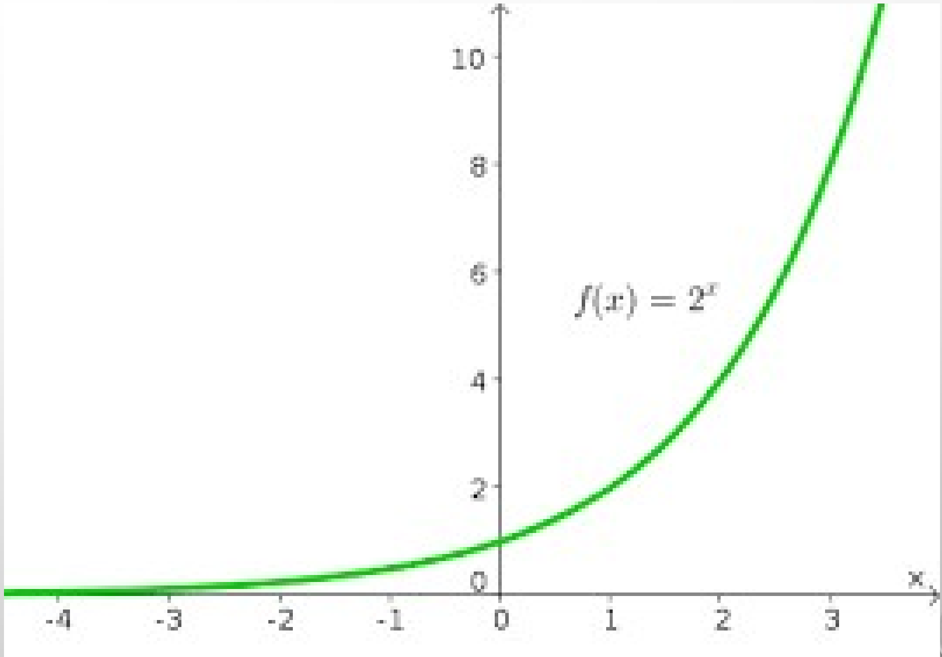
EXPONENTIAL FUNCTIONS

REVIEW

THE BASIC FUNCTION – $f(x) = b^x$

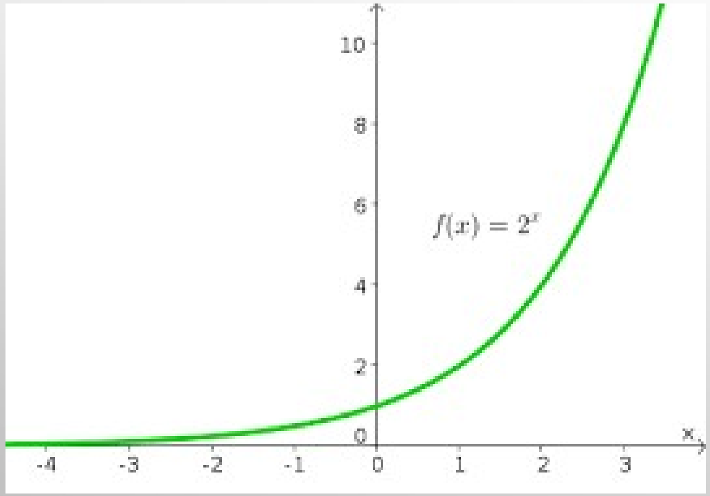
THE GRAPH TO THE RIGHT IS OF A BASIC EXPONENTIAL FUNCTION. THE X IS IN THE EXPONENT. THE 2 IS THE BASE.

NOTICE THAT THE GRAPH APPROACHES, BUT NEVER TOUCHES THE X AXIS. THAT IS THE ASYMPTOTE. THE Y INTERCEPT IS (0,1).

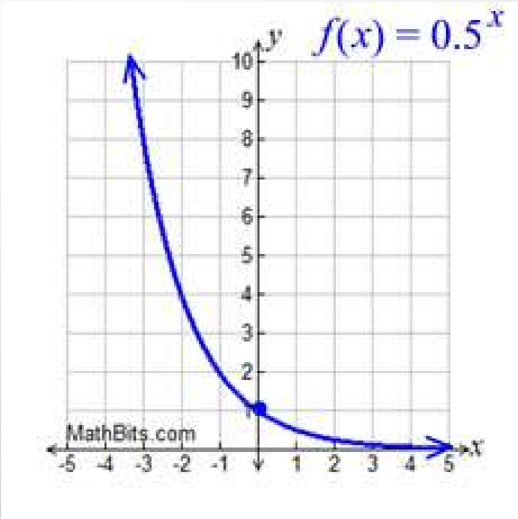


CHARACTERISTICS

GROWTH

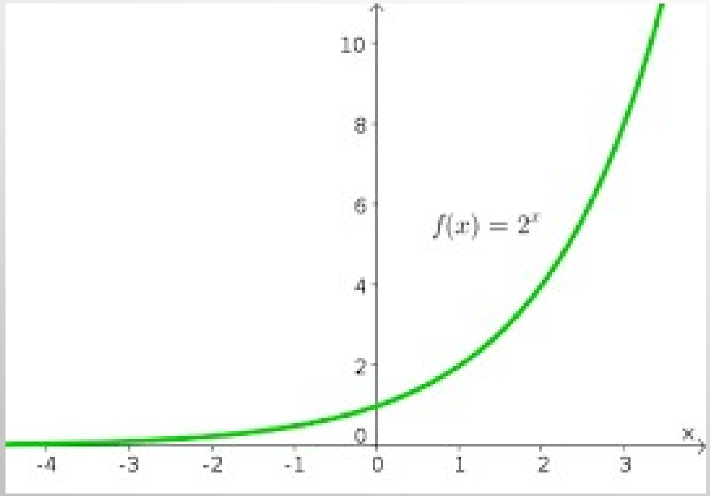


DECAY



CHARACTERISTICS

GROWTH

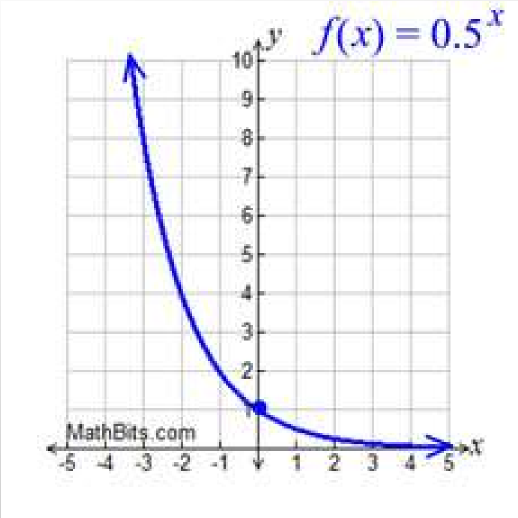


- NOTICE HOW THE BASE IS GREATER THAN 1.
- AS YOU READ THE GRAPH FROM LEFT TO RIGHT, THE Y VALUE IS GETTING LARGER.

CHARACTERISTICS

- NOTICE THE BASE IS SMALLER THAN ONE.
- AS YOU READ THE GRAPH FROM LEFT TO RIGHT THE Y VALUE IS GETTING SMALLER AND SMALLER (BUT WILL STILL NEVER TOUCH THE X AXIS.)
- IF YOU KEEP GIVING AWAY HALF OF YOUR COOKIE, YOU WILL QUICKLY HAVE LITTLE CRUMBS!

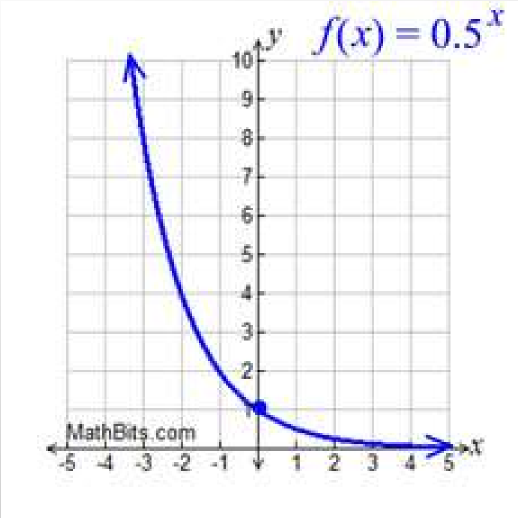
DECAY



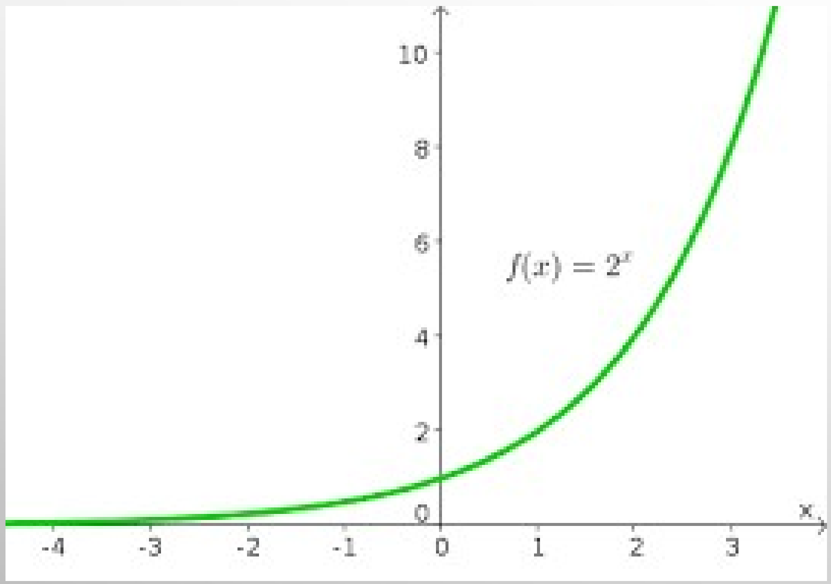
CHARACTERISTICS - ASYMPTOTE

- FOR BOTH THE GROWTH AND DECAY GRAPHS – THEY APPROACH THE X AXIS BUT NEVER TOUCH IT.
- THAT IS THE **ASYMPTOTE**. FOR THE BASIC FUNCTION IT IS ALWAYS Y = 0.
- IT IS AN IMAGINARY BOUNDARY THE GRAPH CAN'T CROSS.

DECAY

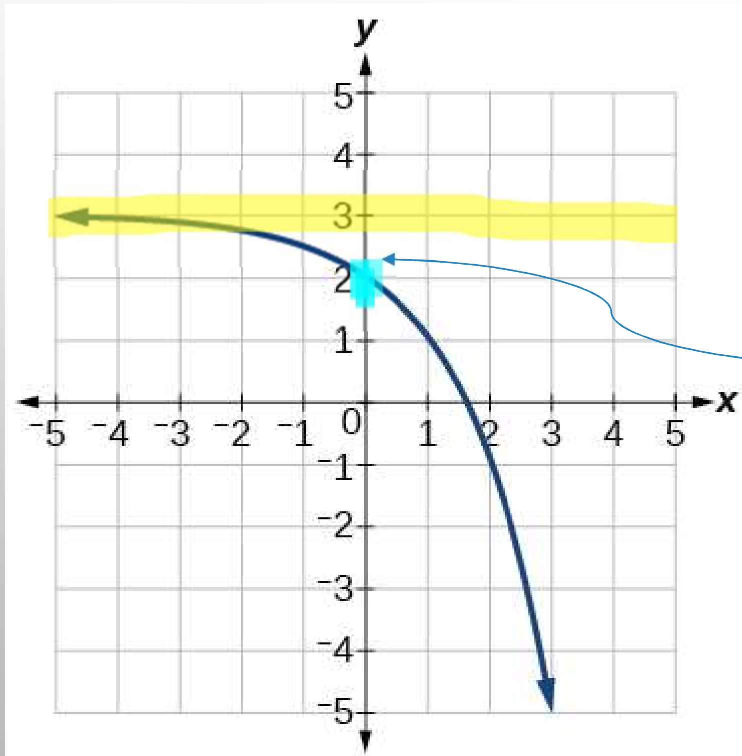


CHARACTERISTICS – DOMAIN, RANGE



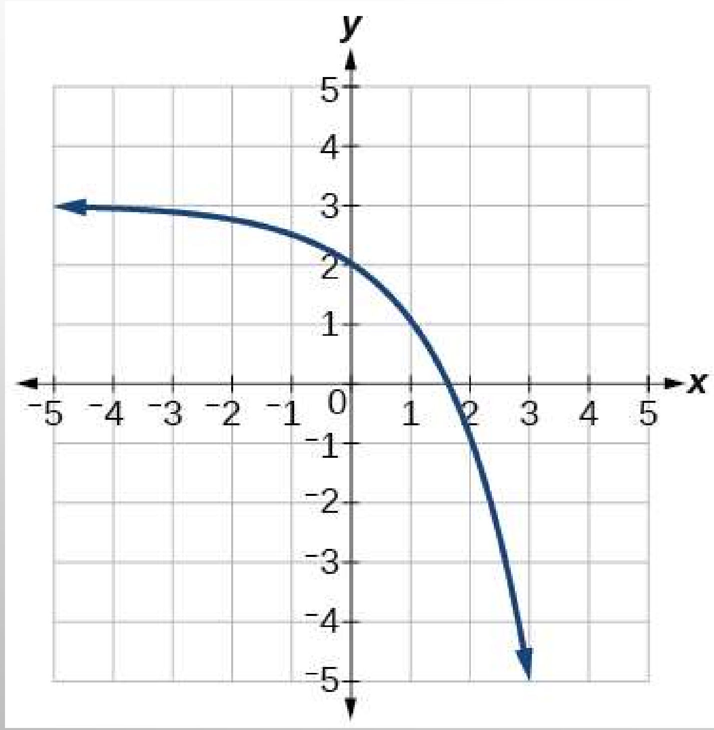
- THE GRAPH WILL GO TO THE LEFT FOREVER AND THE RIGHT FOREVER, SO ALL X VALUES WILL BE COVERED. THE **DOMAIN** WILL BE "ALL REAL NUMBERS"
- THIS GRAPH WILL GO UP FOREVER, BUT IS LIMITED BY THE ASYMPTOTE. THE **RANGE** WILL BE $y > 0$.

CHARACTERISTICS



- FOR THIS GRAPH, THE **ASYMPTOTE** WOULD BE $y = 3$. THE **RANGE** WOULD BE $y < 3$ BECAUSE THE Y VALUES ARE GETTING SMALLER.
- THE **Y INTERCEPT** WOULD BE (0, 2). IT IS THE POINT WHERE THE Y AXIS "CATCHES" THE GRAPH. THE X WILL ALWAYS BE 0 AT THE Y INTERCEPT.

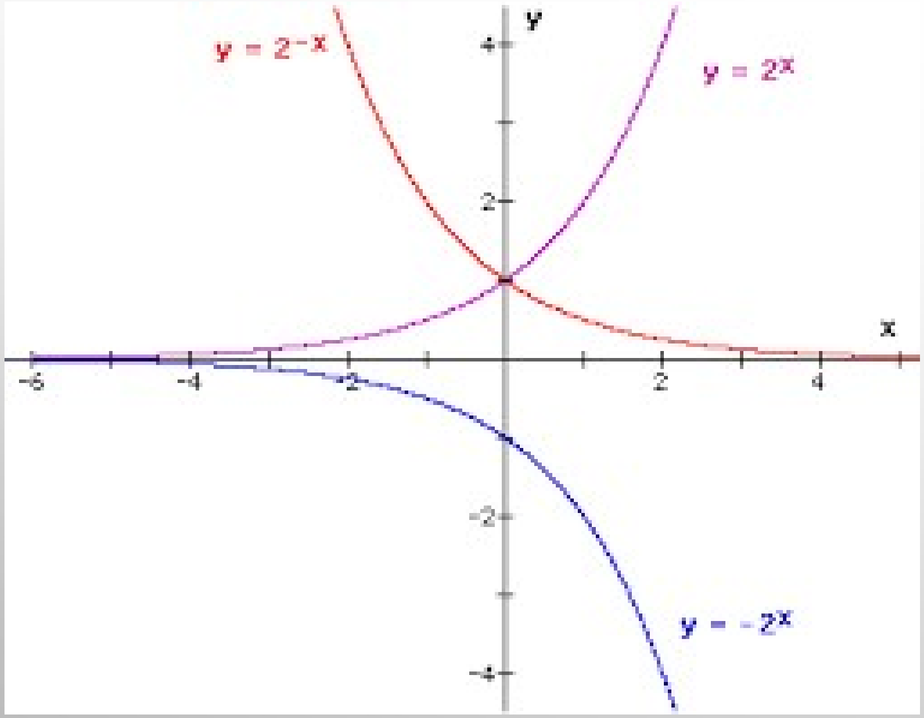
TRANSFORMATIONS



THIS GRAPH IS NOT THE BASIC FUNCTION

- IT HAS BEEN TRANSFORMED, OR MOVED, FROM THE BASIC GRAPH.
- THERE ARE SEVERAL DIFFERENT TRANSFORMATIONS

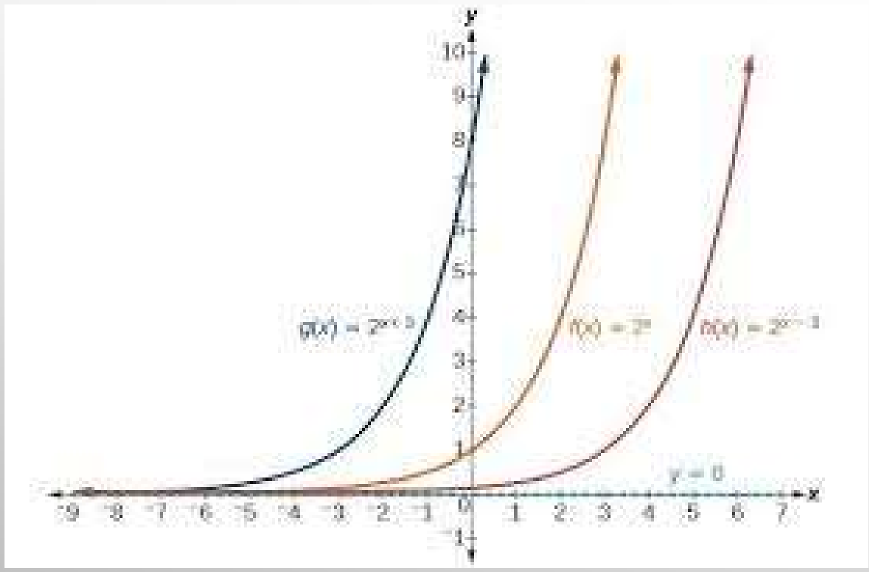
TRANSFORMATIONS – $-a(b)^{-x-h}+k$



REFLECTION

- YOU CAN REFLECT THE GRAPH ACROSS THE X AXIS BY MAKING THE a NEGATIVE
- YOU CAN REFLECT ACROSS THE Y AXIS BY MAKING THE x NEGATIVE
- IN ADDITION TO REFLECTING THE GRAPH, THE a VALUE WILL STRETCH OR SHRINK THE GRAPH.

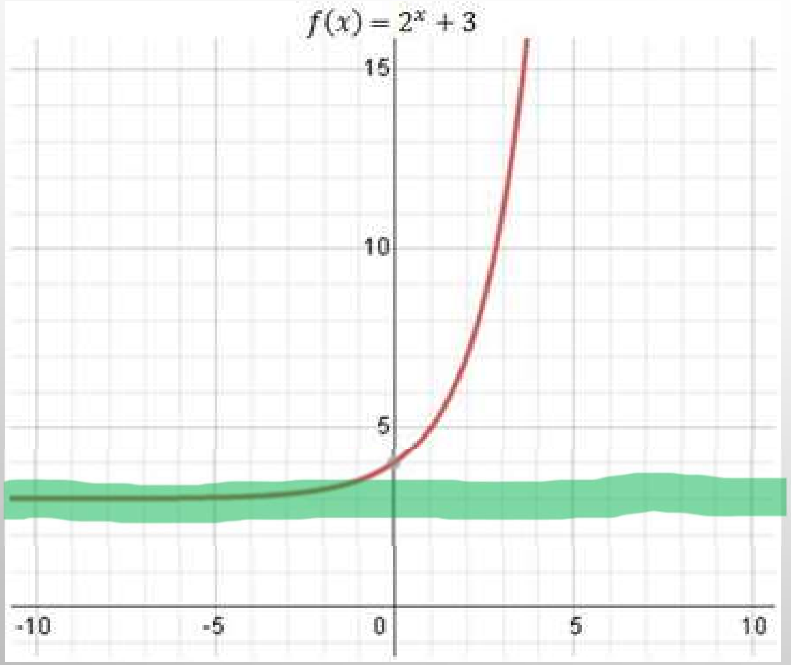
TRANSFORMATIONS - $a(b)^{x-h}+k$



MOVE TO THE RIGHT/LEFT

- THE X AXIS MOVES LEFT AND RIGHT
- THE **h** VALUE IS IN THE EXPONENT WITH THE X
- THE **h** IS THE VALUE THAT MOVES THE GRAPH LEFT AND RIGHT.
- WHEN YOU LEAVE THE EXPONENT, YOU HAVE TO CHANGE SIGNS *(WHEN YOU LEAVE THE HOUSE YOU CHANGE CLOTHES)*
- TO MOVE RIGHT, YOU WOULD HAVE $^{X-3}$
- TO MOVE LEFT, YOU WOULD HAVE $^{X+3}$

TRANSFORMATIONS - $a(b)^{x-h}+k$



MOVE UP/DOWN

- THE Y AXIS RUNS UP AND DOWN
- THE K VALUE MOVES THE GRAPH UP AND DOWN
- IT DOES **NOT** CHANGE SIGN – POSITIVE MOVES UP AND NEGATIVE MOVES DOWN.
- NOTICE THE ASYMPTOTE MOVES WHEN YOU HAVE A K.
- THE NEW ASYMPTOTE FOR THIS GRAPH IS Y = 3
- THE NEW ASYMPTOTE WILL ALWAYS BE YOUR K VALUE.

GRAPH TO EQUATION

- WHEN YOU ARE GIVEN A GRAPH AND NEED TO WRITE AN EQUATION, CHECK FOR A FEW THINGS:
- IS IT GROWTH OR DECAY? HERE Y IS GETTING LARGER, SO IT IS GROWTH. MY BASE WILL BE GREATER THAN 1.
- IS THE GRAPH REFLECTED? HERE IT IS NOT, SO OUR a WILL BE POSITIVE.
- HAS THE ASYMPTOTE MOVED? HERE IT IS 1, SO I KNOW I WILL HAVE A +1 ON MY EQUATION.
- HAS THE Y INTERCEPT CHANGED (NORMALLY AT (0,1))? HERE THE GRAPH HAS BEEN MOVED LEFT, SO I KNOW I WILL HAVE AN X + NUMBER IN THE EXPONENT.

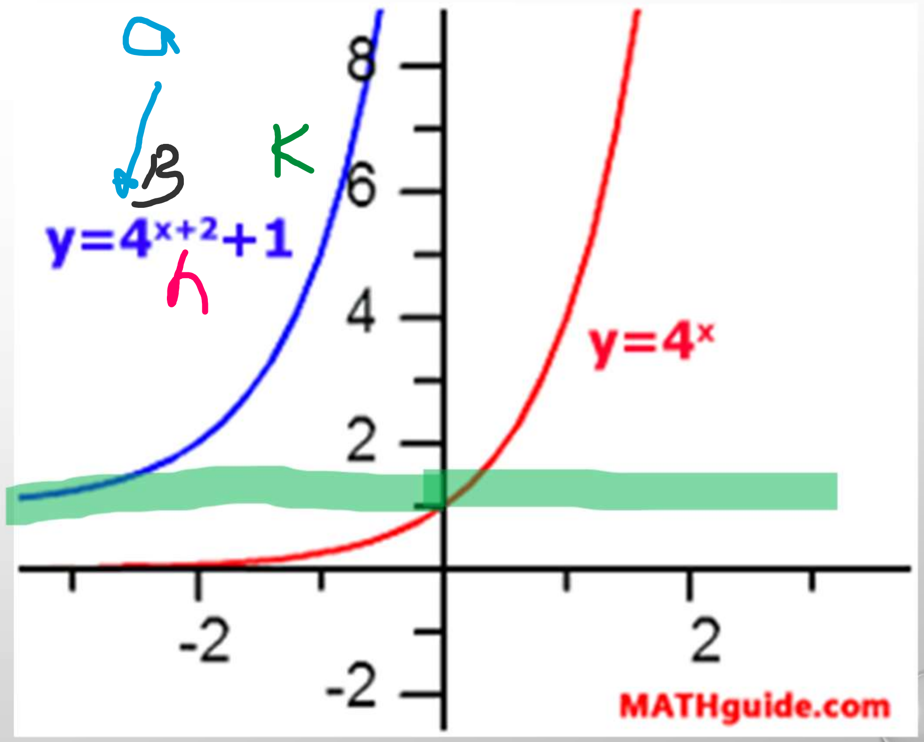
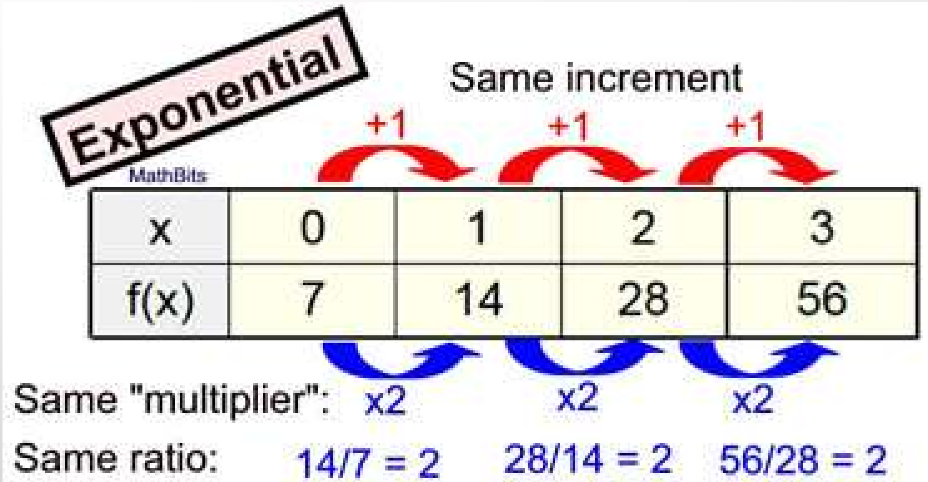


TABLE TO EQUATION

- WHEN YOU ARE GIVEN A TABLE AND NEED TO WRITE AN EQUATION, CHECK FOR A FEW THINGS:
- WHAT IS MY INITIAL VALUE?
  - REMEMBER THE X VALUES ARE USUALLY ORDINAL: 1^{ST} TERM, 2^{ND} TERM, ETC.
  - THE <u>Y OR f(x) VALUES ARE THE VALUE OF THE TERM</u>
  - HERE THE FIRST OR INITIAL TERM IS 7 – THAT IS MY a VALUE
- WHAT IS THE RATIO?
  - DIVIDE THE 2^{ND} TERM BY THE 1^{ST}
  - THEN DO A CHECK CALCULATION, EITHER
    - DIVIDE THE 3^{RD} TERM BY THE 2^{ND}, OR
    - MULTIPLY THE 2^{ND} TERM BY THE RATIO
  - THE **RATIO IS YOUR BASE OR b VALUE**
- IN THIS EXAMPLE $f(x) = 7(2)^x$



APPLICATIONS

$$a(b)^x$$

- NO PERCENTAGE
- CONSTANT RATIO
  - DOUBLE, TRIPLE, HALF LIFE
- a: STARTING POINT, INITIAL VALUE
- b: THE RATIO
  - DOUBLE (2)
  - HALF LFIE (1/2)
- x : TIME
- EXAMPLE: MY KNOWLEDGE OF EXPONENTIAL FUNCTIONS DOUBLES WITH EVERY HOUR I STUDY. IF I START GETTING 6 PROBLEMS RIGHT, HOW MANY WILL I GET AFTER STUDYING FOR 3 HOURS?

APPLICATIONS

$$a(1 \pm r)^t$$

- PERCENTAGES!
- NO COMPOUNDING
- **a**: STARTING POINT, INITIAL VALUE
- USE ADDITION IF THE AMOUNT IS INCREASING; SUBTRACTION IF THE AMOUNT IS DECREASING OR GOING DOWN.
- **r**: THE RATE
  - YOU MUST CHANGE THE % INTO A DECIMAL
  - MOVE THE DECIMAL POINT TO THE LEFT 2 PLACES
  - 5% = .05
- **t** : TIME
- EXAMPLE: I EAT 10% OF THE CHEEZ ITS IN THE BOX FOR EVERY HOUR I WORK AT MY DESK. IF I START WITH 300 CHEEZ ITS, HOW MANY WILL BE LEFT AFTER I WORK 5 HOURS?

APPLICATIONS

$$P(1+\frac{r}{n})^{nt}$$

- PERCENTAGES AND **COMPOUNDING** *(THEY WILL USE THAT WORD)*
- P: STARTING POINT, INITIAL VALUE
- r: THE RATE
  - YOU MUST CHANGE THE % INTO A DECIMAL
  - MOVE THE DECIMAL POINT TO THE LEFT 2 PLACES
  - *5% = .05*
- n: THE NUMBER OF TIMES IT IS COMPOUNDED PER YEAR
  - ANNUALLY – 1; SEMI ANNUALLY – 2; QUARTERLY – 4; MONTHLY - 12
- t : TIME; THE NUMBER OF YEARS
- EXAMPLE: IF I DEPOSIT $200 IN AN ACCOUNT WHERE 3% INTEREST IS COMPOUNDED MONTHLY, HOW MUCH WILL I HAVE IN 50 YEARS?

GEOMETRIC SEQUENCES – EXPLICIT FORMULA

- $a_n = a_1 \ r^{n-1}$

- a_n = THE TERM YOU ARE LOOKING FOR AT THE "N[TH]" PLACE; THINK OF IT LIKE YOUR "Y"
- **r** = THE RATIO; THE NUMBER YOU ARE MULTIPLYING OR DIVIDING BY
- a_1 = THE FIRST TERM OF THE SEQUENCE; IT IS IN PLACE "1"
- $^{n-1}$ = THE TERM BEFORE THE TERM YOU ARE LOOKING FOR

GEOMETRIC SEQUENCES – EXPLICIT FORMULA

- $a_n = a_1 \; r^{\,n-1}$

- $a_7 = (-6) \; (-6)^{\,7-1}$

- **WRITE THE <u>EXPLICIT RULE</u> FOR THE GEOMETRIC SEQUENCE BELOW. THEN FIND THE 7TH TERM**
- -6, 36, -216, 1296 …
- $r = 36/-6 = -6$
- $a_1 = -6$
- $n = 7$